

Exercise 28Calculate y' .

$$y = (\cos x)^x$$

Solution

Bring the variable out of the exponent by introducing a logarithm.

$$\begin{aligned}y' &= \frac{d}{dx}[(\cos x)^x] \\&= \frac{d}{dx} \left[e^{\ln(\cos x)^x} \right] \\&= \frac{d}{dx} \left[e^{x \ln(\cos x)} \right] \\&= e^{x \ln(\cos x)} \cdot \frac{d}{dx} [x \ln(\cos x)] \\&= e^{x \ln(\cos x)} \cdot \left\{ \left[\frac{d}{dx}(x) \right] \ln(\cos x) + x \left[\frac{d}{dx} \ln(\cos x) \right] \right\} \\&= e^{\ln(\cos x)^x} \cdot \left\{ (1) \ln(\cos x) + x \left[\frac{1}{\cos x} \cdot \frac{d}{dx}(\cos x) \right] \right\} \\&= (\cos x)^x \cdot \left\{ \ln(\cos x) + x \left[\frac{1}{\cos x} \cdot (-\sin x) \right] \right\} \\&= (\cos x)^x [\ln(\cos x) - x \tan x]\end{aligned}$$